1 a $\Phi_B = B l x$

x increases in time as $x = x_0 + vt$

$\Phi_B = B l (x_0 + v_0 t)$

$\mathcal{E} = - \frac{d\Phi_B}{dt} = - Blv.$

b $\mathcal{E} = I R$

$I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$

B points out of the paper; flux increases and nature abhorse this change.
Need a field to counteract; induced current will run cw

c Constant velocity means no total force on rod.

$F_{external} = F_{lorentz} = I l B$

B is up; I is down in rod

$F_{lorentz}$ points to the ~~right~~ left

The force to keep constant motion is equal to $F_{lorentz}$ and points right

1c continued

$$P_{ext} = F_{ext} v \quad (\text{power!})$$
$$= I l B v$$
$$= \frac{\varepsilon}{R} l B v = \frac{(Blv)^2}{R}$$

1d. Electric power

$$P_{elec} = I^2 R = \left(\frac{Blv}{R}\right)^2 R = \frac{(Blv)^2}{R}$$

note $P_{elec} = P_{ext}$

2 a $$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t}(V/d) = \frac{1}{d}\frac{\partial V}{\partial t} = \frac{1}{10^{-3}} 100 \ V/(ms)$$
$$= 10^5 \ V/(ms)$$

b $$\vec{J}_D = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$= 8.85 \ 10^{-12} \ 10^5 \ A/m^2 = 8.85 \ 10^{-7} \ A/m^2$$

$$I_D = \int \vec{J} \cdot d\vec{a} \qquad \vec{J} \parallel d\vec{a}$$
$$I_D = J \pi r_0^2 = J \pi (d/2)^2 = 2.5 \cdot 10^{-9} \ A.$$

2 C

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{D\,encl.}$$

Amperian loop = circel with radius r and surface πr^2

$$I_{D\,encl} = J_D \pi r^2$$

$$2\pi r B = \mu_0 J_D \pi r^2$$

$$B = \frac{\mu_0 J_D r}{2} = \frac{\mu_0 \varepsilon_0}{2} r \left|\frac{\partial \vec{E}}{\partial t}\right|$$

$$= \frac{r}{2c^2} \left|\frac{\partial E}{\partial t}\right| = 5.6 \cdot 10^{-15}\,T.$$

3 a.

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_e}{\varepsilon_0} \qquad \vec{\nabla}\times\vec{E} = -\mu_0 J_m - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = \mu_0 \rho_m \qquad \vec{\nabla}\times\vec{B} = \mu_0 J_e + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

with

$$\vec{\nabla}\cdot J_m + \frac{\partial \rho_m}{\partial t} = 0$$

$$\vec{\nabla}\cdot\vec{J_e} + \frac{\partial \rho_e}{\partial t} = 0$$

3 b $\Phi_m = \int \vec{B} \cdot \vec{da}$

$$\mathcal{E} = -\frac{d}{dt}\Phi_m = -L\frac{\partial I}{\partial t}$$

Assume that current is zero, we can find the change in the current as:

$$\Delta I = \Delta \Phi_m / L$$

If there is a magnetic monopole it has a charge q_m.

$\vec{B}$ is similar to $\vec{E}$ for electric charges:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2}$$

$$\Phi_m = \int \vec{B} \cdot \vec{da} = \frac{\mu_0}{4\pi}\frac{q_m}{r^2} 4\pi r^2$$

This flux emitted by q_m has to pass through the loop, if the monopole flies through

$\Delta \Phi_m = \mu_0 q_m$. Thus: $\Delta I = \frac{\mu_0 q_m}{L}$

4.a.

Use the law of Faraday

$$\oint_C \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int_S \vec{B} \cdot \hat{n} \, da$$

path　　　　　　　　surface.

The system has cylinder symmetry thus with $\vec{B}$ along $\hat{z}$ axis, the $\vec{E}$ field runs in circles.

Magnetic field for solenoid $r < R$.

$$\vec{B} = \frac{\mu_0 N I}{l} \hat{z}$$

flux through surface with radius $r < R$

$$\Phi_B = \int_S \vec{B} \cdot \hat{n} \, da$$

$$= \frac{\mu_0 N I}{l} \pi r^2$$

with $I = I_0 \sin(\omega t)$.

Therefore $\frac{d}{dt} \Phi_B = \frac{\mu_0 N I_0}{l} \pi r^2 \omega \cos(\omega t)$

4 continued

a) For $r < R$ the loop around the surface da is given as

$$\oint_C \vec{E} \cdot \vec{dl} = 2\pi r E$$

Therefore we find:

$$2\pi r |E| = -\frac{\mu_0 N_0 \pi r^2}{l} \omega I_0 \cos(\omega t)$$

solve for $|E| = -\dfrac{\mu_0 N r \omega I_0}{2l} \cos(\omega t)$.

b) For $r > R$ ($B = 0$ $r > R$)

$$\oint_C \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int_S \vec{B} \cdot \vec{da}$$

$$= -\frac{d}{dt} \left[\pi R^2 \frac{\mu_0 N I}{l} \right]$$

$$2\pi r |E| = -\frac{\mu_0 N \pi R^2}{l} \omega I_0 \cos(\omega t)$$

$$|E| = -\frac{\mu_0 N R^2 \omega I_0}{2 r l} \cos(\omega t)$$

5a. No Lorentz contraction

$E'^{x}_{x} = E_x.$

b $B_x = \mu_0 n I.$ $\gamma^2 = \frac{1}{(1-v}$

$n' = \gamma n.$

$I' = \frac{I}{\gamma}$

$B'_x = B_x$

c $\vec{E}' \cdot \vec{B}' = E_x' B_x' + E_y' \cdot B_y' + \cancel{B}'_z B_z$

$= E_x B_x + \gamma^2 E_y B_y - \cancel{\gamma^2 v B_z B_y}$

$+ \cancel{\gamma^2 \frac{v}{c^2} E_z B_y} - \gamma^2 \frac{v^2}{c^2} B_z E_z$

$+ \gamma^2 E_z B_z - \gamma^2 \frac{v^2}{c^2} B_y E_y - \cancel{\gamma^2 \frac{v}{c^2} E_z E_y} + \cancel{\gamma^2 v B_y B_z}$

$= E_x B_x + (1 - \frac{v^2}{c^2}) \gamma^2 E_y B_y + (1 - \frac{v^2}{c^2}) \gamma^2 E_z B_z$

$= E_x B_x + E_y B_y + E_z B_z.$

d. wave equation: ~~D~~ $\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$

M2: $\vec{\nabla} \times \vec{E} = - \frac{\partial B}{\partial t}$

$\vec{E} = E_0 \sin(-\omega t + kx) \hat{y}$

$\vec{B} = \frac{E_0}{c} \sin(-\omega t + kx) \hat{z}$

5 d continued

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix}$$

$$= \frac{\partial}{\partial x} E_y \hat{z} = k E_0 \cos(kx - \omega t) \hat{z}$$

But $\frac{\partial}{\partial t} \vec{B} = - \omega \frac{E_0}{c} \cos(kx - \omega t) \hat{z}$

wave equation : $k E_0 \cos(\ ) \hat{z} = \omega \frac{E_0}{c} \cos\ ) \hat{z}$

$$k = \frac{\omega}{c} \quad OK.$$

M4 $\frac{\partial}{\partial x} \vec{B} = - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$k \frac{E_0}{c} \cos(\quad) = \mu_0 \varepsilon_0 \omega E_0 \cos(\quad)$$

$$\frac{k}{c} = \mu_0 \varepsilon_0 \omega = \frac{\omega}{c^2} \qquad k = \frac{\omega}{c} \quad OK.$$

e y direction ($\vec{E}$ vector)

f. $E_y' = \gamma (E_y - v B_z)$

$v = c \quad \beta = 1 \quad \gamma \to \infty$!

5 f continued.

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}} = \frac{1}{(1-\beta)^{1/2}(1+\beta)^{1/2}}$$

$$E_y' = \frac{1-\beta}{(1-\beta)^{1/2}(1+\beta)^{1/2}} E_y = \frac{(1-\beta)^{1/2}}{(1+\beta)^{1/2}} E_y = 0.$$

$$B_{\cancel{z}}' = \gamma\left(B_z - \frac{\beta}{c}E_y\right) = \gamma\left(\frac{E_0}{c} - \frac{\beta}{c}E_0\right)$$

$$= \gamma(1-\beta)\frac{E_0}{c}$$

$$= \frac{(1-\beta)}{(1-\beta)^{1/2}(1+\beta)^{1/2}}\frac{E_0}{c}$$

$$= \frac{(1-\beta)^{1/2}}{(1+\beta)^{1/2}}\frac{E_0}{c}$$

$$\beta = 1 \quad B_z' = 0$$

~~Both~~

$$B_y' = 0 \qquad E_z' = 0$$

$$E_x' = 0 \qquad B_z' = 0$$

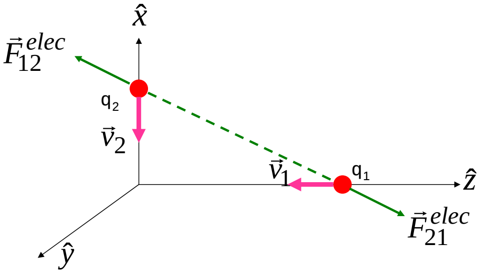
$$\vec{E} = \vec{0} \text{ and } \vec{B} = \vec{0} \;!$$

Third law of Newton
Force between two charges

Consider two identical point charges, which are forced to move towards each other. Is the third law of Newton valid for this case?

Electric forces: $\vec{F}^{elec}_{12} = -\vec{F}^{elec}_{21}$

Action = - reaction



Third law of Newton

Force between two charges

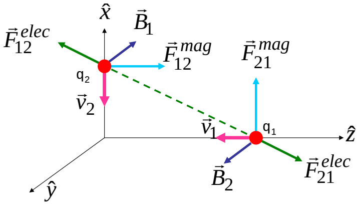
Consider two identical point charges, which are forced to move towards each other. Is the third law of Newton valid for this case?

Magnetic forces: $\vec{F}_{12}^{mag} \neq -\vec{F}_{21}^{mag}$

$\vec{F}_{12}^{mag} \quad // \quad \hat{z}$

$\vec{F}_{21}^{mag} \quad // \quad \hat{x}$

Action $\neq$ - reaction



Be careful in chapter 10 we will see the details!!